微積分期末考 (2003 Spring) 系級: 學號: 姓名:

1. (a) Find the derivative of $f(x,y,z) = x^3 - xy^2 - z$ at $P(1,1,0)$ in the direction of $\vec{v} = 2\vec{i} - 3\vec{j} + 6\vec{k}$. (10分)
(b) In what direction does f increase most rapidly at $P(1,1,0)$ and what is the rate of change in the direction. (10分)

2. Show that $\lim_{(x,y)\to(0,0)} \frac{x^2y}{x^4+y^2}$ does not exist. (10分)

3. Find the tangent plane and normal line of the surface $x^2+y^2+z-9=0$ at the point $P(1,2,4)$. (20分)

4. Find the saddle points of $f(x,y) = x^2 - y^2 - 2x + 4y + 6$. (10分)

5. Use the Method of Lagrange Multiplies to find the maximum and minimum values of $f(x, y, z) = x - 2y + 5z$ on the sphere $x^2 + y^2 + z^2 = 30$. (20分)

6. Evaluate $\int\int_R (y - 2x^2)\, dA$ where R is the region bounded by the square $|x| + |y| = 1$. (10分)

7. Find the area of the region cut from the first quadrant by the cardioid $r = 1 + \cos\theta$. (10分)

8. Applying a transformation to evaluate (10分)

$$\int_0^1 \int_0^{1-x} \sqrt{x+y}\,(y-x)^2\, dy\, dx.$$